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continued fractions and infinite series. Approximations¹ are obtained far beyond any conceivable practical need and mathematicians come to suspect that π is not only irrational but not an algebraic irrational.

The third period may be called the *critical* period, since it was devoted to "critical investigations of the true nature of the number π itself, considered independently of mere analytical representation."² The period extends from the middle of the eighteenth century late into the nineteenth century when the transcendence of π is finally definitely established.

William Jones³ seems to have been the first to make use of the symbol π with its present special significance, but its permanent use as such was chiefly due to the influence of Euler.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

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PROBLEMS FOR SOLUTION.

2773. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

Point out the fallacy in the proof following the problem:

In the triangle $A_1B_1C_1$ let M be a point such that the sum of the distances from it to the sides is a maximum; also, let $A_2B_2C_2$ be a triangle formed by drawing lines through the vertices A_1 , B_1 , and C_1 parallel to their opposite sides. Then the sum of the distances from M to the sides of the triangle $A_2B_2C_2$ is a minimum.

Proof.—Because the sides of the two triangles are parallel in pairs, the sum of the distances from a variable point P in triangle $A_1B_1C_1$ to the six sides of the two triangles is constant. Now by hypothesis M is a point for which one part of this constant sum is a maximum, and hence it follows that the other part is a minimum.

2774. Proposed by FRANK IRWIN, University of California.

Evaluate the circulants

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & 1 & 2 & \cdots & n-2 & n-1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & 4 & \cdots & n & 1 \end{vmatrix}, \quad \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & a_4 & \cdots & a_n & a_1 \end{vmatrix},$$

where, in the latter, $a_1, a_2, \dots, a_n$ form an arithmetical progression.

¹ A condensed table of approximations of π as determined by various men is given by J. W. L. Glaisher, *Messenger of Mathematics*, vol. 2, pp. 122–128, and in vol. 3, pp. 45–46, some corrections are made of the table given on p. 122 of vol. 2. The approximation was carried to 707 places by William Shanks in 1873 (*Proceedings of the Royal Society of London*, vol. 21, p. 318 and vol. 22, p. 45). A considerable list of approximations is given by Ball, *loc. cit.*, 4th ed., pp. 250–261, 5th ed., pp. 296–306.

² Hobson, *l. c.*, p. 12.

³ *Synopsis Palmariorum Matheseos*, London, 1706, pp. 243, 263, *et seq.* Cited by Ball, *loc. cit.*, 4th ed., p. 250, 5th ed., p. 296, and by others. Concerning its early use by others, see article "Sur le premier emploi du symbole π pour 3.14159..." by G. Eneström, *Bibliotheca Mathematica*, 1889, p. 28.

2775. Proposed by H. T. BURGESS, University of Wisconsin.

Solve in finite form, if possible, the differential equation

$$\frac{d^2y}{dt^2} + ay \frac{dy}{dt} + by = g,$$

when a and b are arbitrary constants and $g = 32.16$. When $t = 0$, $y = 0$, $dy/dt = 0$.

As solutions have not been received for more than 130 problems proposed since January, 1913, the number of *new* problems proposed each month is to be considerably reduced and *old* problems are to be repropoed. Solutions are desired for the following problems proposed before January, 1918:

Algebra—406, 411, 416, 417, 461, 481, 494.*Geometry*—442, 446, 455, 463, 470, 472, 476, 477, 478, 499, 501, 510, 519, 523.*Calculus*—348, 349, 353, 406, 415, 429, 432, 434, 436.*Mechanics*—287, 291, 300, 308, 309, 313, 315, 332, 343, 344, 351.*Number Theory*—198, 201, 202, 205, 231, 232, 234, 238, 245, 247, 263, 266, 270, 272, 273, 274, 275.**430 (Algebra) [March, 1915]. Proposed by V. M. SPUNAR, Chicago, Illinois.**

Solve the following equations both algebraically and graphically:

$$x^y + y^x = xy, \quad x^x + y^y = x + y.$$

339 (Calculus) [June, 1913]. Proposed by T. H. GRONWALL, New York, N. Y.To show that for any real value of x

$$\left| \frac{d^n}{dx^n} \left(\frac{\sin x}{x} \right) \right| \leq \frac{1}{n+1}, \quad \text{and} \quad \left| \frac{d^n}{dx^n} \left(\frac{1 - \cos x}{x} \right) \right| \leq \frac{1}{n+1}.$$

340 (Calculus) [June, 1913]. Proposed by C. N. SCHMALL, New York, N. Y.

A pencil of parallel rays of light is incident upon a lens whose faces have the radii r_1 , r_2 , respectively. Show that the distance of the principal focus from the center of the first face of the lens will be a maximum or a minimum when

$$\frac{r_1}{r_2} = \frac{(\mu - 1)^{1/2}}{1 + (\mu - 1)^{1/2}},$$

where μ has its usual meaning.**272 (Mechanics) [February, 1913]. Proposed by J. F. LAWRENCE, Stillwater, Okla.**

A perfectly rough circular cylinder is fixed with its axis horizontal. A sphere is placed on it in a position of unstable equilibrium, and projected with a given velocity parallel to the axis of the cylinder. If the sphere be slightly disturbed in a horizontal direction perpendicular to the direction of the axis of the cylinder, determine at what point the sphere will leave the cylinder.

277¹ (Mechanics) [June, 1913]. Proposed by W. J. GREENSTREET, Editor of the Mathematical Gazette, England.

Around a smooth fixed circular pulley is wound a massless inextensible string, and straight portions go to two free ends A and B to which masses are fastened. The mass at A is initially projected perpendicular to the string while the other is initially at rest. The length of the straight portion to the first mass is initially l and subsequently is r . Find the velocity of the second mass at that moment.

¹ Incorrectly numbered 272 when first proposed.